Investigation of New Time-Frequency Analyses for Acoustic Transients

By

William J. Williams
Electrical Engineering and Computer Science Department
Systems Division

University of Michigan
Ann Arbor, MI 48109

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1. INTRODUCTION

Signals of practical interest often do not conform to the requirements of realistic application of Fourier principles. The approach works best when the signal of interest is composed of a number of discrete frequency components so that time is not a specific issue. (e.g. a constant frequency sinusoid) or somewhat paradoxically, when the signal exists for a very short time so that its time of occurrence is considered to be known (e.g. an impulse function). Much of what we are taught implies that signals that can not be satisfactorily represented in these ways are somehow suspect and must be forced into the mold or abandoned.

It has been quite difficult to handle nonstationary signals such as chirps satisfactorily using conceptualizations based on stationarity. The spectrogram represents an attempt to apply the Fourier transform for a short-time analysis window, within which it is hoped that the signal behaves reasonably according to the requirements of stationarity.Many real-world signals, particularly biological signals, do not conform to these requirements. By moving the analysis window along the signal, one hopes to track and capture the variations of the signal spectrum as a function of time. The well-known spectrogram is an example of such an approach. The spectrogram has many useful properties including a well developed general theory. It has been used with great success for many years and has provided many useful insights into biological phenomena, particularly speech. The spectrogram often presents serious difficulties when used to analyze rapidly varying signals, however. If the analysis window is made short enough to capture rapid changes in the signal it becomes impossible to resolve frequency components of the signal which are close in frequency during the analysis window duration. On the other hand, if the time window is made long enough to permit good frequency resolution, it is difficult to determine where, in time, that the various frequency components act. There are many assumptions in conventional engineering analysis which allow us to view signals from an idealized viewpoint. The Fourier transform is defined to be:

$$Z(\omega) = F[z(t)] = \int_{-\infty}^{\infty} z(t)e^{-j\omega t}dt \tag{1.1}$$

and its inverse,

$$z(t) = F^{-1}[Z(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z(\omega)e^{j\omega t}d\omega \tag{1.2}$$

This very familiar transform is certainly well-known to the reader. However, one seldom questions the integral limits. Everyone knows that it is not possible to obtain $z(t)$ in a practical sense. How could one know $z(t)$ for all time? Likewise, it is impossible to know, in a practical sense, what $Z(\omega)$ is for all frequencies. If we

have a function which expresses $z(t)$ or $Z(\omega)$, then there is no problem. However, we may often neglect to apply this thinking when dealing with real-world signals. The Fourier transform essentially implies that one does not need to worry about time after the transform is applied. Time has been integrated out of the picture. All one cares about is the frequency content of the signal. There is no attention to *when* the signal components of different frequency act. Likewise, when the inverse transform is obtained, one is supposed to have no interest in the frequency of the various components of the signal, $z(t)$. It is tacitly assumed that the frequency components of the signal are eternal and not changing with time. This is the basis of the Fourier series which is a weighted sum of sin and cosine terms. Figure 1 illustrates this. There are three components with different frequencies present in $z(t)$. The magnitude of $Z(\omega)$ is also shown. Next, these same components are windowed in time and combined to provide a sequential combination rather than a simultaneous combination of these frequency components. The results of this experiment are shown in Figure 2 [1]

One can see that there is no evidence of the difference in the time action of the sinusoidal components in the transform domain. Certainly, the simultaneous sinwaves yield a sharper spectrum due to the fact that they are longer in duration. Where and when they act is unclear from the spectrum, however. This justifies the need for joint time-frequency representations. The spectrogram has long been a useful tool in time-frequency analysis. The basic idea behind the spectrogram is to assume that the signal is stationary or quasi-stationary over a limited time window. This time window is moved along the signal and a time-indexed spectrum is computed. The continuous formulation is as follows from the Short Time Fourier Transform or STFT:

$$STFT_z(t,\omega) = \int z(\tau)h(\tau - t)e^{-j\omega\tau}d\tau \tag{1.3}$$

and, then,

$$SP_z(t,\omega) = |STFT_z(t,\omega)|^2 \tag{1.4}$$

where $h(t)$ is the window function. The spectrogram suffers from a window tradeoff condition which is often known as the Uncertainty Principle. Long time windows provide good frequency resolution, but poor time resolution. Short time windows provide good time resolution, but poor frequency resolution. One must make a choice.

The spectrogram has been a very useful tool in time-frequency analysis. However, it has several serious liabilities and limitations that we will cover in detail in

[1] The actual computation was done using a 512 point FFT and the sample rate assumed was 1 Hz.

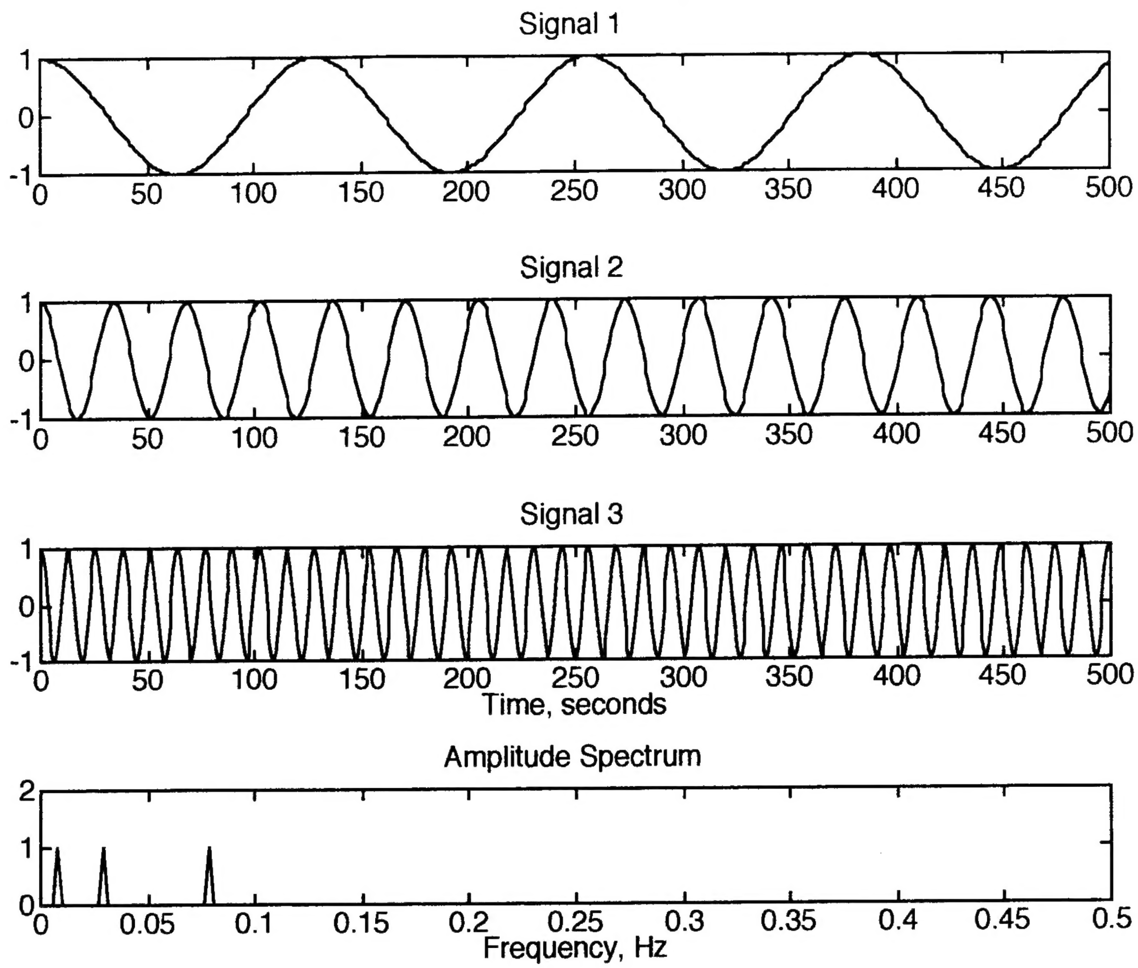


Figure 1: Three sinwaves simultaneous in time and the amplitude spectrum of their sum

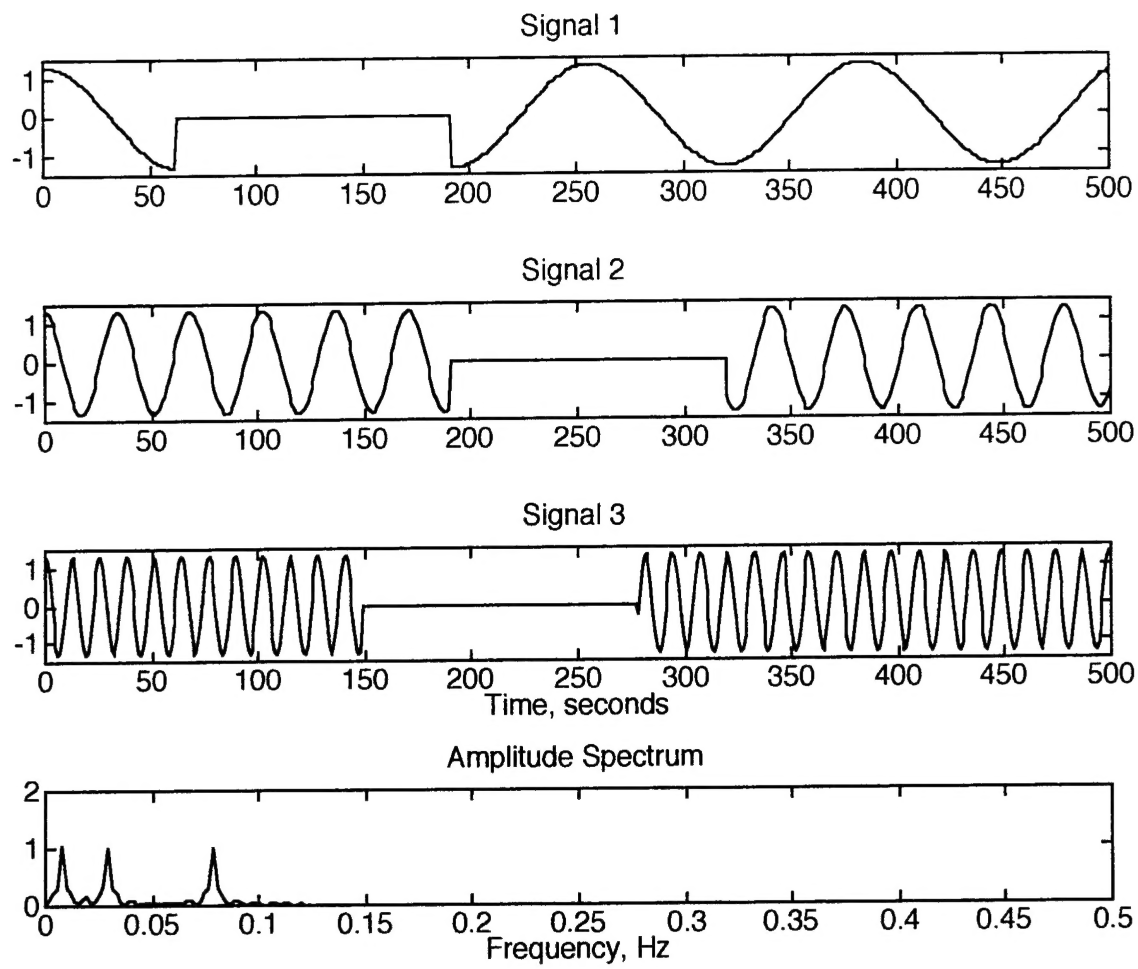


Figure 2: The signals from Figure 1 with gaps and the amplitude spectrum of their sum

this chapter. More recent time-frequency developments have provided useful and interesting alternatives to spectrograms.

The Wigner distribution (WD) has been employed as an alternative to overcome the liabilities and limitations of the spectrogram. The WD was first introduced in the context of quantum mechanics [19] and revived for signal analysis by Ville [17].The WD has many important and interesting properties[4, 5, 6]. It provides a high resolution representation in time and in frequency for a non stationary signal such as a chirp. In addition, the WD has the important property of satisfying the time and frequency marginals in terms of the instantaneous power in time and energy spectrum in frequency. However, its energy distribution is not non-negative and it often possesses severe cross terms, or interference terms, between components in different t-f regions, potentially leading to confusion and misinterpretation. An excellent discussion on the geometry of interferences has been provided by Hlawatsch and Flandrin [10, 11, 12].

Both the spectrogram and the WD are members of Cohen's class of distributions [7]. Cohen has provided a consistent set of definitions for a desirable set of t-f distributions which has been of great value in guiding and clarifying efforts in this area of research. Cohen's Class of Distributions is defined to be:

$$C_z(t,\omega;\phi) = \iiint e^{j(-\theta t-\tau\theta+\omega u)}\phi(\theta,\tau)z(u+\tau/2)z^*(u-\tau/2)\,du\,d\tau\,d\theta \qquad (1.5)$$

where $z(t)$ is the time signal, $z^*(t)$ is its complex conjugate, and $\phi(\theta,\tau)$ is the **kernel** of the distribution.[2]

A recent comprehensive review by Cohen[8] provides an excellent overview of time-frequency distributions and recent results using them. This paper addresses a specific subset of t-f distributions belonging to Cohen's class. These are the time shift and frequency shift invariant t-f distributions. For these distributions a time shift in the signal is reflected as an equivalent time shift in the t-f distribution and a shift in the frequency of the signal is reflected as an equivalent frequency shift in the t-f distribution. The spectrogram, the WD and the RID all have this property. Different distributions can be obtained by selecting different kernel functions in Cohen's class. Boashash has compared the performances of several time-frequency distributions in terms of resolution [2]. Desirable properties of a distribution and associated kernel requirements have been extensively investigated by Claasen and Mecklenbrauker [4, 5, 6].

The Wigner distribution is general expressed as:

[2]The range of integrals is from $-\infty$ to ∞ throughout this paper unless otherwise indicated.

$$W_z(t,\omega) = \int z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j\omega\tau}d\tau \qquad (1.6)$$

or, in its dual form as:

$$W_z(t,\omega) = \frac{1}{2\pi}\int Z^*(\omega+\frac{\theta}{2})Z(\omega-\frac{\theta}{2})e^{-j\theta t}d\theta \qquad (1.7)$$

The Wigner distribution often provides high time and frequency resolution results for simple monocomponent signals. However, if $z(t) = a(t) + b(t)$, then the Wigner distribution consists of four components, $W_{aa}(t,\omega)+W_{ab}(t,\omega)+W_{ba}(t,\omega)+W_{bb}(t,\omega)$. If, due to symmetry, $W_ab(t,\omega)$ and $W_ba(t,\omega)$ combine, then an interference term or cross-term which has twice the amplitude of $W_aa(t,\omega)$ and $W_bb(t,\omega)$ if $W_aa(t,\omega)$ are equal in amplitude.

More recently, Choi and Williams introduced a new distribution having an exponential-type kernel [3], which the called the Exponential Distribution or ED. This new distribution overcomes several drawbacks of the spectrogram and WD, this distribution provides high resolution with suppressed interferences [3, 13, 14]. This distribution has been called the Choi-Williams distribution, first by Cohen and subsequently by a number of other investigators. We shall prefer to refer to this specific example as the ED and the general class of Reduced Interference Distributions as RIDs.

Another new time-frequency distribution has also received a lot of attention in recent years. This is the Cone Kernel Distribution or the ZAM distribution introduced by Zhao, Atlas and Marks [21]. The ZAM is spectrogram-like in some aspects, but it overcomes several of the liabilities of the spectrogram and offers high resolution along with sharp time delineation and good frequency resolution of segmented sinewaves.

Time-frequency distributions (TFDs) have been termed so due to their similarities and analogies to probabilistic concepts. Some prefer to call them time-frequency representations (TFRs) to highlight the fact that they are not really distributions in the probabalisitic sense. In this work we will use the term TFRs in general, though distribution will be retained when refering to specific members of Cohen's class. There are a number of TFRs which have recently arisen or evolved and are based on the elements of one or more of the TFRs just mentioned. These TFRs will be discussed later in this chapter. One particularly useful method of viewing TFRs will be discussed next. This is the Reduced Interference Distribution or RID.

2. THE REDUCED INTERFERENCE DISTRIBUTION

2.1 Ambiguity Function Relationships

The key to understanding t-f relationships and manipulations is a thorough understanding of the ambiguity domain. Let $Z(\omega)$ be the FT of the signal $z(t)$:

$$Z(\omega) = F[z(t)] = \int f(t)e^{-j\omega t}dt \tag{2.1}$$

and

$$z(t) = F^{-1}[Z(\omega)] = \int Z(\omega)e^{j\omega t}d\omega \tag{2.2}$$

Let $R_z(t, \tau)$ be the *instantaneous autocorrelation* of a complex signal $z(t)$, defined as:

$$R_z(t, \tau) = z(t + \tau/2)z^*(t - \tau/2) \tag{2.3}$$

where f^* denotes the complex conjugate of f. The Wigner distribution of $z(t)$ is defined as the Fourier Transform (FT) of $R_f(t, \tau)$ with respect to the lag variable τ.

$$W_z(t, \omega) = F_\tau[z(t + \tau/2)z^*(t - \tau/2)] = F_\tau[R_z(t, \tau)] \tag{2.4}$$

Similarly, but with a different physical meaning, the symmetrical ambiguity function (AF) is defined as the Inverse Fourier transform (IFT) of $R_z(t, \tau)$ with respect to the first variable.

$$A_z(\theta, \tau) = F_t^{-1}[z(t + \tau/2)z^*(t - \tau/2)] = F_t^{-1}[R_z(t, \tau)] \tag{2.5}$$

Thus, $W_z(t, \omega)$ and $A_z\theta\tau)$ are related by the two-dimensional (2-D) FT.

$$W_z(t, \omega) = \iint A_z(\theta, \tau)e^{-j(t\theta + \omega\tau)}d\theta d\tau \tag{2.6}$$

These relationships may be combined with Equation 1.1 to show that $C_z(t, f; \phi)$ may be found by

$$C_z(t, \omega, \phi) = \iint \phi(\theta, \tau)A_z(\theta, \tau)e^{-j(t\theta + \omega\tau)}d\theta d\tau \tag{2.7}$$

Thus, while the Wigner distribution may be found from the symmetric ambiguity function by means of a double Fourier transform, any member of Cohen's class of distributions may be found by first multiplying the kernel, $\phi\theta\tau)$ by the symmetric ambiguity function and then carrying out the double Fourier transform. The generalized ambiguity function, $\phi\theta\tau)A_z\theta\tau)$ [9] is a key concept in t-f which aids one in

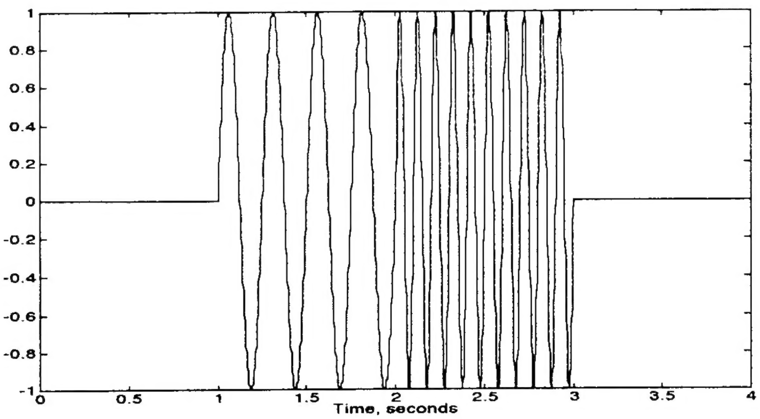


Figure 3: Test signal used to evaluate the time-frequency distributions

clearly seeing the effect of the kernel in determining $C_z(t, \omega; \phi)$. A test signal for evaluating some of the properties of time-frequency distributions is introduced at this point. The test signal consists of two sinsoidal segments of differing time and frequency placement. This signal is shown in Figure 3. The Wigner distribution and the ambiguity function of the two sinusoids displaced in time and frequency are shown in Figure 4

Thus, if the Wigner kernel is multiplied by the ambiguity function, the ambiguity function not altered. The Wigner time-frequency result is shown in Figure 5.

It can be shown[8] that the kernel of the spectrogram is the Wigner distribution of the time window itself. Since the time window is gaussian in this case, the kernel is a two dimensional gaussian function of v and τ. Figure6 shows the spectrogram kernel and the result of its effect on the ambiguity function.

It can readily be seen that the spectrogram kernel radically alters the ambiguity function. This has a marked effect on the time-frequency distribution as well The spectrogram kernel low-pass filters the ambiguity function. The resulting spectrogram is shown in Figure 7.

The kernel for the WD is unity, so the generalized ambiguity function is identical to the ambiguity function and its t-f representation (the double Fourier transform) preserves both the auto-terms and the cross-terms. The kernels of the spectrogram and the RID emphasize the auto-terms and demphasize the cross terms, but in very different ways.

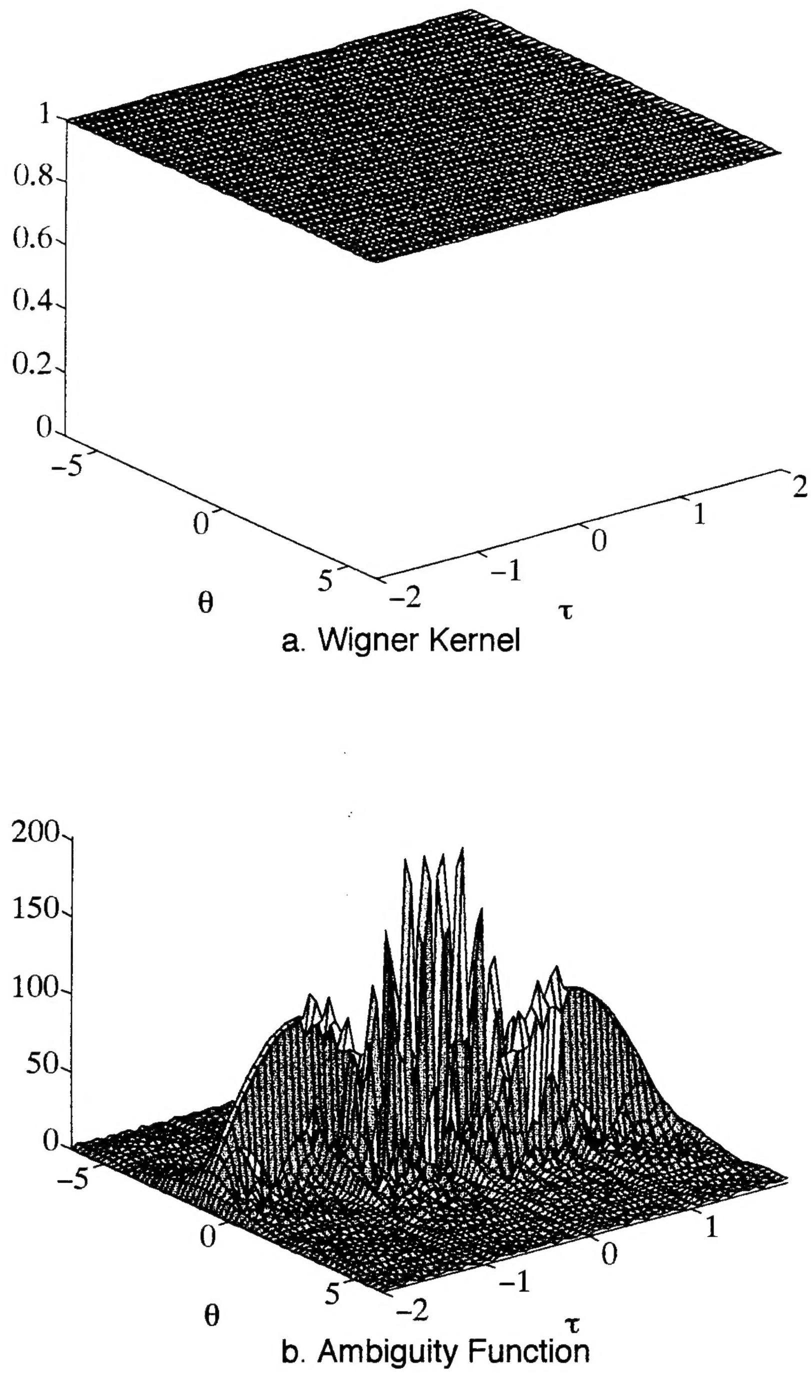


Figure 4: Wigner distribribution kernel (a.) and the ambiguity function (b.)

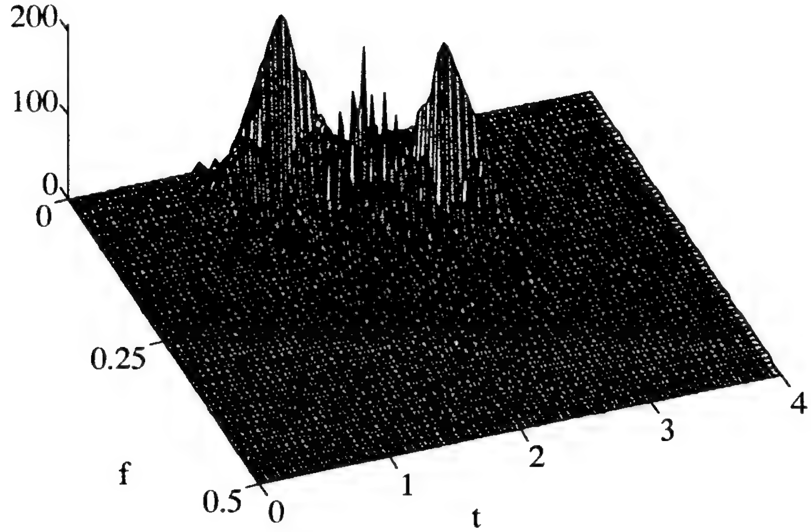


Figure 5: Wigner distribution time-frequency result

2.2 The Exponential Distribution

The distributions just discussed each has properties that are valuable under certain conditions. The ED (aka Choi-Williams distribution) is an attempt [3] to improve on the WD. It has a kernel $\phi(\theta,\tau)$=exp$(-\frac{\theta^2\tau^2}{\sigma})$, and it proves to be quite effective in suppressing the interferences while retaining high resolution.Its performance has been compared with those of the spectrogram and the WD in a variety of environments [13,14]. The σ parameter may be varied over a range of values to obtain different trade-offs between cross-term suppression and high auto-term t-f resolution. In fact, as σ becomes very large the ED kernel approaches the WD kernel. This provides the best resolution but the cross-terms become large and approach WD cross-terms in size. Unfortunately, however, in a strict sense, this distribution violates the support properties, but does satisfy them with small error. This is not a very important practical issue, since a window can easily be imposed when the t,τ form of the ED kernel is convolved in time with the local autocorrelation prior to Fourier transforming with respect to τ to obtain the $ED(t,\omega)$ form of the distribution, thus insuring that the support properties are exactly satisfied. The windowed ED RID ambiguity plane results are shown in Figure 2.2. It can be seen that the RID kernel captures the central portion of the

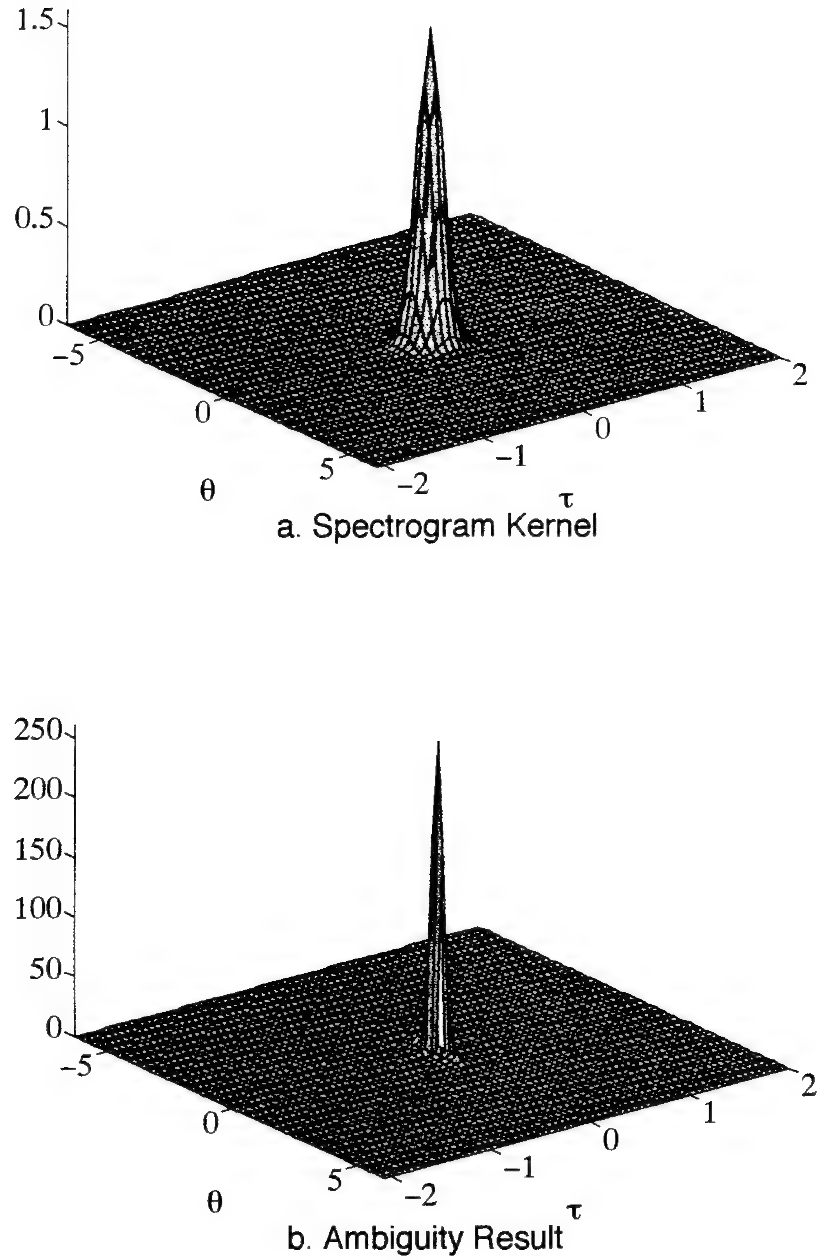


Figure 6: Spectrogram kernel (a.) and the resulting altered ambiguity function (b.)

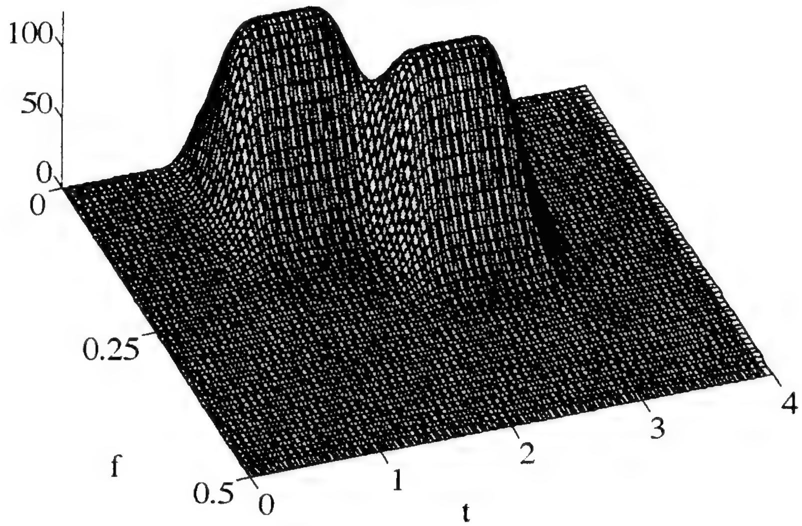


Figure 7: Spectrogram time-frequency result

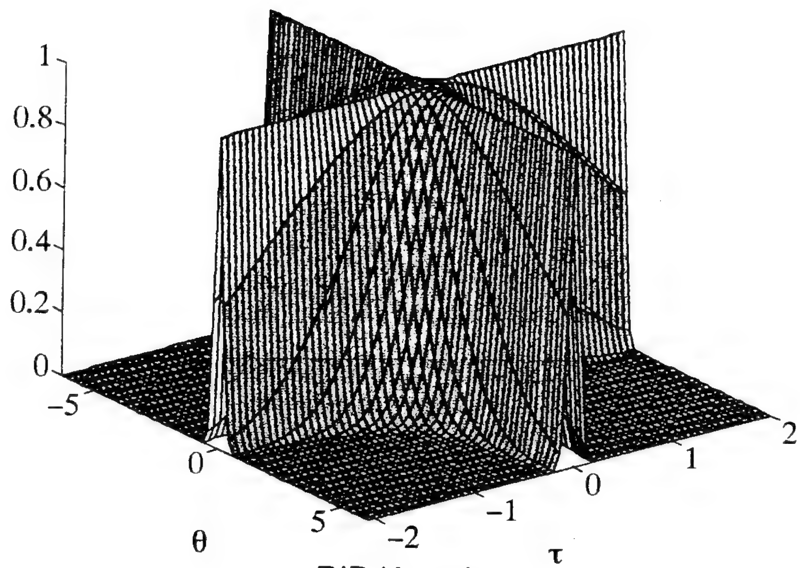
ambiguity function and rejects the outlying cross-terms.

The RID kernel keeps much more of the ambiguity function. The offending interference terms are essentially excluded here. The resulting time-frequency distribution is shown in Figure 9.

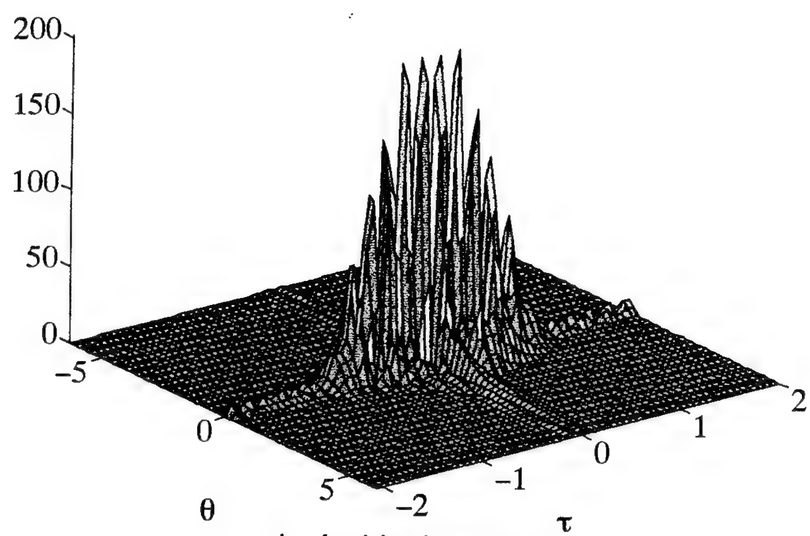
2.3 Zhao-Atlas-Marks

The Zhao-Atlas-Marks (ZAM) [21] distribution or cone-kernel distribution had distinctively different motivation than the RID. The formulation was motivated by the phenomenon of lateral inhibition in the auditory system. The ZAM kernel adheres to the requirement which guarantee that the time-support property is met. Its kernel is presented in Table 1, but, unlike the Wigner distribution, the spectrogram and the RID, the ambiguity plane provides a generally confusing picture of how the kernel manifests itself in producing good results. The original form of the ZAM simply enforced the time support property on the local-autocorrelation. Its formulation is:

$$C_{ZAM}(t,\omega) = \int \int_{t-\frac{|\tau|}{2}}^{t+\frac{|\tau|}{2}} z(u+\frac{1}{2}\tau)z^*(u-\frac{1}{2}\tau)\exp^{-j\omega\tau}\, du d\tau \tag{2.8}$$



a. RID Kernel



b. Ambiguity Result

Figure 8: RID kernel (a.) and the resulting altered ambiguity function (b.)

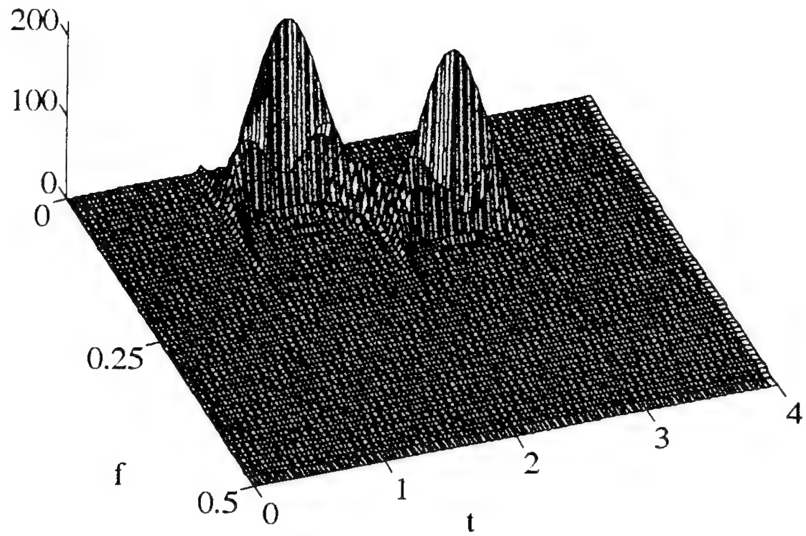


Figure 9: RID time-frequency result

The ZAM result for the test signal is shown in Figure 10.

It can be seen that the ZAM produces a nice result, resolving the sinwave segments well in time and in frequency. In contrast to the RID, the ZAM in general places the interference terms at approximately the same time-frequency locations as the auto-terms for such signals. There are some clear differences in RID and ZAM that should be taken into consideration, however. These differences will be discussed further as the tools for understanding time-frequency distributions are further developed.

2.4 Kernel Selection for RID

A more formal description of RIDs is appropriate at this point. Requirements for the RID and the RIDs properties are quite similar to the WD. Once these properties are laid out, it will be possible to compare and contrast different distributions with much greater ease. The properties of the WD are investigated in [[8, 4, 5, 6]]. RID requirements and properties will be discussed in comparison with the WD.The unity value of the WD kernel guarantees the desirable properties of the WD. How-

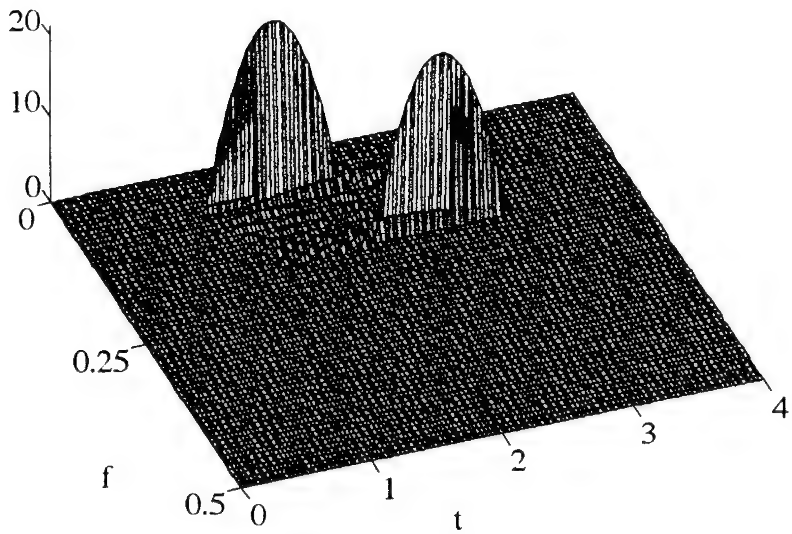


Figure 10: ZAM time-frequency result

ever, it is not necessary to require the kernel to be unity for all ω and τ in order to maintain most of its desirable properties. It is sufficient to insure that the kernel is unity along $\omega = 0$ and $\tau = 0$ and that the kernel is such that $\phi^*(\theta, \tau) = g(-\theta, -\tau)$, the later property insuring realness. The RID kernel is cross shaped and acts as a low pass filter in both θ and τ. Returning to Figure 6 one can see that the spectrogram suffers from poor resolution, whereas the WD and the RID exhibit good resolution and support properties Figures 4 and 2.2. However, the WD also exhibits interference terms. The spectrogram has the virtue of suppressing cross-terms as does the RID and has the further advantage of being non-negative which is not the case for the WD and the RID. The RID possesses almost all of the desirable properties of the WD except for its unitary property, $|\phi(\theta, \tau)|$=1 for all θ, τ.

It is quite desirable for a distribution to possess the time and frequency support property. This property insures that the distribution does not extend beyond the support of the signal in time or the support of its Fourier transform in frequency. One can see in Figure 6 that the spectrogram violates this property rather badly. The time and frequency support property may be maintained for the RID by

P0.	nonnegativity : $C_z(t,\omega;\phi) \geq 0 \ \forall t,\omega$
Q0.	$\phi(\theta\tau)$ is the ambiguity function of some function $w(t)$.
P1.	realness : $C_z(t,\omega;\phi) \in R$
Q1.	$\phi(\theta,\tau) = \phi^*(-v,-\tau)$
P2.	time shift : $s(t) = z(t-t_0) \Rightarrow C_s(t,\omega;\phi) = C_z(t-t_0,\omega;\phi)$
Q2.	$\phi\theta\tau)$ does not depend on t.
P3.	frequency shift : $s(t) = z(t)e^{j\omega_0 t} \Rightarrow C_s(t,\omega;\phi) = C_z(t,\omega-\omega_0;\phi)$
Q3.	$\phi(\theta,\tau)$ does not depend on ω.
P4.	time marginal : $\int W_z(t,\omega)d\omega = z(t)z^*(t)$
Q4.	$\phi\theta 0) = 1 \ \forall\theta$
P5.	frequency marginal : $\int C_z(t,\omega;\phi)dt = Z(\omega)Z^*(\omega)$
Q5.	$\phi(0,\tau) = 1 \ \forall\tau$
P6.	instantaneous frequency : $\frac{\int f C_z(t,\omega;\phi)d\omega}{\int C_z(t,\omega;\phi)d\omega} = \omega_i(t)$
Q6.	Q4 and $\frac{\partial\phi\theta\tau)}{\partial\tau}\|_{\tau=0} = 0 \ \forall v$
P7.	group delay : $\frac{\int t C_z(t,\omega;\phi)dt}{\int C_z(t,\omega;\phi)dt} = t_g(\omega)$
Q7.	Q5 and $\frac{\partial\phi\theta\tau)}{\partial\theta}\|_{\theta=0} = 0 \ \forall\tau$
P8.	time support : $z(t) = 0$ for $\|t\| > t_c \Rightarrow C_z(t,\omega;\phi) = 0$ for $\|t\| > t_c$
Q8.	$\psi(t,\tau) \triangleq \int \phi\theta\tau)e^{-j2\pi vt}dv = 0$ for $\|\tau\| < 2\|t\|$
P9.	frequency support : $Z(\omega) = 0$ for $\|\omega\| > \omega_c \Rightarrow C_z(t,\omega;\phi) = 0$ for $\|\omega\| > \omega_c$
Q9.	$\int \phi(\theta,\tau)e^{j\omega\tau}d\tau = 0$ for $\|\theta\| < 2\|\omega\|$
P10.	Reduced Interference
Q10.	$\phi(\theta,\tau)$ is a 2-D low pass filter type.

Table 1: Distribution properties and associated kernel requirements.

insuring that

$$\psi(t,\tau) = \int \phi(\theta,\tau)e^{-j\theta t}d\theta = 0 \ if \ |\tau| < 2|t|. \tag{2.9}$$

This forms a "cone shaped" region in t,τ. The WD obviously satisfies this support property since the Fourier transform of unity is an impulse function, clearly staying within the t,τ limits. The form of the kernel in θ,ω is also cone shaped, insuring the frequency support property. Zhao, Atlas and Marks [21] suggest a cone shaped kernel for nonstationary signal analysis, but they impose restrictions such that time support only is insured. The ED can be brought into the RID requirements by imposing a RID window as suggested above. The RID is not a totally new distribution since the Born-Jordon kernel [8] , $\phi(\theta,\tau) = sinc(\theta\tau)$ meets all of the RID requirements.

Distribution	$\phi(\theta,\tau)$	P0	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Wigner	1		x	x	x	x	x	x	x	x	x	
Rihaczek	$e^{j\theta\tau/2}$			x	x	x	x			x	x	
Re{Rihaczek}	$\cos(\theta\tau/2)$		x	x	x	x	x	x	x	x	x	
Exponential (ED)	$e^{-\theta^2\tau^2/2\sigma}$		x	x	x	x	x	x	x			x
Spectrogram	$A_w\theta\tau)$ of a window $w(t)$	x	x	x	x	x						x
Born-Jordan	$\frac{\sin(\theta\tau/2)}{\theta\tau/2}$		x	x	x	x	x	x	x	x	x	x
Windowed-ED	$e^{-v^2/2\sigma} * W(v)\|_{v=\theta\tau}$		x	x	x	x	x	x	x	x	x	x
Cone (ZAM)[3]	$g(\tau)\|\tau\|\frac{sin(a\theta\tau)}{a\theta\tau}$		x	x	x	x				x		x

Table 2: Comparison among several distributions in terms of their properties.

Table 1 summarizes desirable properties (P) and related kernel qualifications (Q) of distributions. No known, practical distribution is able to meet all of these requirements.

Table 2 illustrates how several common distributions satisfy the desirable properties. Two distributions whose kernels meet the RID requirements (Born-Jordon and windowed ED) are included.

The windowed-ED and the Born-Jordan (aka Cohen's Born-Jordan) distributions are members of the RID class of distributions. That the Born-Jordan distribution was a member of the RID class was discovered when the RID was defined [20, 15]. The RID is a very general concept which can be used to design a large number of distributions with desirable characteristics.

2.5 Design Procedures for Effective RID kernels

There is much more that can be done in terms of kernel design. It is possible to bring much of the work that has been done on windows and digital filters to bear in designing effective RID kernels [15]. The starting point for the kernel design is to consider a primitive function, h(t). This function is designed to have unit area, i.e, $\int h(t)dt = 1$. It is designed to be symmetrical, .i.e, $h(-t) = h(t)$. It is limited such that $h(t) = 0$ for $|t| > 1/2$. The design is such that h(t) is tapered so that it has little high frequency content, that is, it is a low pass filter. Then the kernel is

$$\phi(\theta, \tau) = H(\theta\tau) \tag{2.10}$$

where $H(\theta)$ is the Fourier transform of h(t). It can be shown that $H(\theta\tau)$ satisfies the RID requirements. Furthermore the desirable characteristics of h(t) that are required have been the subject of intensive study in terms of windows in

time domain terms or filters in frequency domain terms. All of this knowledge can be used to select effective RID kernels.

2.6 Discrete RID kernels

Time-frequency distributions are presented in a continuous form for theoretical development and discussion of properties. However, one usually wishes to utilize a discrete form of the distribution for computational convenience using a digital computer. claasen and Mecklenbrauker [5] develop discrete forms of the Wigner distribution,

$$W_z(n,\omega) = 2\sum_{k=-\infty}^{\infty} e^{-j2\omega k} z(n+k).z^*(n-k) \tag{2.11}$$

If discrete values of the local autocorrelation, $R_z(n,m)$, were available for all integer n and m, then it would be logical to express the discrete form of the Wigner distribution as the discrete-time Fourier transform (DTFT) of $R_z(n,m)$, or

$$W_z(n,\omega) = \sum_{m=-\infty}^{\infty} e^{-j2\omega m} R_z(n,m) \tag{2.12}$$

Notice that as k takes on the values 0,1,2 .. the discrete local autocorrelations $R_z(n,0) = z(n)z^*(n), R_z(n,2) = z(n+1)z^*(n-1), R_z(n,4) = z(n+2)z^*(n-2)$ are evaulated. Discrete values two samples appart are thus correlated. Local autocorrelation values for odd integer spacings are not available, so the discrete form of the Wigner distribution is formed from the evenly spaced correlation values. This means that the local autocorrelation is undersampled by a factor of two compared to the sequence z(n) and aliasing may occur in the discrete WD if z(n) were not sampled at twice the Nyquist frequency for a real valued z(n).The analytic form of the signal presents no problem with aliasing however, since only half the period of the DTFT spectrum is occupied. If aliasing is a problem with the discrete WD and not with the original sequence z(n), then additional points may be interpolated for z(n) to fill in the missing correlation values required to form the discrete WD.

Except for potential aliasing problems, the discrete form of the Wigner distribution enjoys many of the desirable properties of the continuous form and suffers from similar limitations. Practical computation requires a finite length sequence of z(n) values. A discrete time-discrete frequency version of the WD may be expressed as [5] a discrete-time windowed version of the infinite length sequence form, with interpolation of the odd indexed values of the local autocorrelation if required. The resulting discrete distribution is termed the "pseudo Wigner distribution (PWD)". The PWD may be computed by efficient means involving FFTs. Cohen's review [8] mentions several such important efficient computational algorithms.

Requirements for discrete forms of the RID are similar to those of the discrete WD. The discrete RID may be formed by

$$RID_z(n,\omega) = \sum_{m=-\infty}^{\infty} R_z(n,m) *_n \psi(n,m) e^{-\omega m} \tag{2.13}$$

where

$$\phi(m,\omega) = \sum_{n=-\infty}^{\infty} \psi(n,m) e^{-\omega n} \tag{2.14}$$

is the discrete RID kernel.

The discrete RID may thus be convieniently formed by obtaining the local autocorrelation $R_z(n,m)$, convolving it with $\psi(n,m)$ along n and DTFTing the result with respect to m. A partially discrete form of the ED called the Running Windowed Exponential Distribution (RWED) has been suggested [3]. The RWED retains the continuous form in the RWED and the limiting forms (when the kernel approaches an impulse function) must be evaluated as special cases. A fully discrete form of the kernel is more desirable. A very convenient discrete RID kernel has been discovered based on the binomial distribution [20]. The form of the kernel is

$$\psi(n,m) = \frac{1}{2^m}\tbinom{m}{k}\delta(n-m+k) \tag{2.15}$$

The correlation shift index , m, is assumed to take on the values $-\infty..-1,0,1,..\infty$ and the time shift index , n is assumed to take on the values $-\infty,..-1,-.5,0,.5,1,..\infty$. It can be shown that the signal structure of the discrete local autocorrelation and the discrete form of the kernel can be easily formulated to include the half-integers [20, 16].

There are no doubt many ways of realizing discrete RIDs. Amin [1] has suggested applying singular value decomposition to the full rank description of a kernel matrix in the time and lag variables. The aim is to find a rank one kernel estimator which is closest to the full rank kernel description so that the kernel may be more simply expressed and decrease computational complexity. Amin approximated the ED kernel using this technique and obtained fairly nice looking t-f plots for pairs of chirps. However, the kernel was not well approximated by the rank one estimate. The question remains as to how much the desirable properties of the distribution are eroded by this approximation. Increasing the rank of the approximation quickly improves the kernel representation (Amin-personal communication). However, this also increases the computational complexity. Practical RID computation in many real-world situations may require parallel VLSI realizations or dedicated DSP modules.

3. Applications of RID

The RID has been examined in a number of acoustic transient applications. A particular effort has been carried out at Woods Hole Oceanographic Instititon in collaboration with Dr. William Watkins and his group. The RID technique has been ported to their analysis computer and is in regular use in their applications. His group has found that RID "often reveals to the eye what the ear hears". The following example illustrates the effectiveness of RID in revealing the fine structure of dolphin whistles. Figure 11 shows the RID of three signals. The first signal is a dolphin click. Next, in the center is the same signal scaled by two and shifted in time. The signal to the right of the display is the original signal shifted to twice the center frequency of the original signal and shifted in time. One can see that these signals have complex time-frequency structures. There are two vertical impulsive chirps and two horizontal tone-like components as well as a moderately down-sloping chirp. The basic shape is preserved in each case. The time and frequency shifted version of the click looks just like the original. The scaled (compressed by a factor of 2) signal in the middle preserves the relationships between the components. When a signal is compressed in time it expands by the same factor in frequency under RID analysis, just as the properties of the Fourier transform suggest. It should be noted that the spectrogram will distort the scaled signal and the wavelet transform will distort the frequency shifted signal. The RID has desirable attributes of both.

4. Conclusions

The RID and other new time frequency concepts, including wavelet transforms, have the potential of revealing many new phenomena and changing the way we view and think about signals. These are only beginnings. The coming years are sure to be exciting as these new concepts are developed and applied. There will be disappointments and trade-offs as well. There is much more to do on the theoretical, computational and application fronts. The new advances in computer technology will no doubt advance apace with these new concepts and make possible applications of concepts that were so far removed from that possibility that they were not even contemplated until recently. The RID has several advantages over both the spectrogram and the wavelet transform. The spectrogram and the wavelet transform both suffer from an uncertaintly region imposed by their windows. In the spectrogram, this window is uniform over time and frequency, whereas for the wavelet transform, the window is long in time and narrow in frequency for low frequencies and the reverse for high frequencies. The RID can resolve both

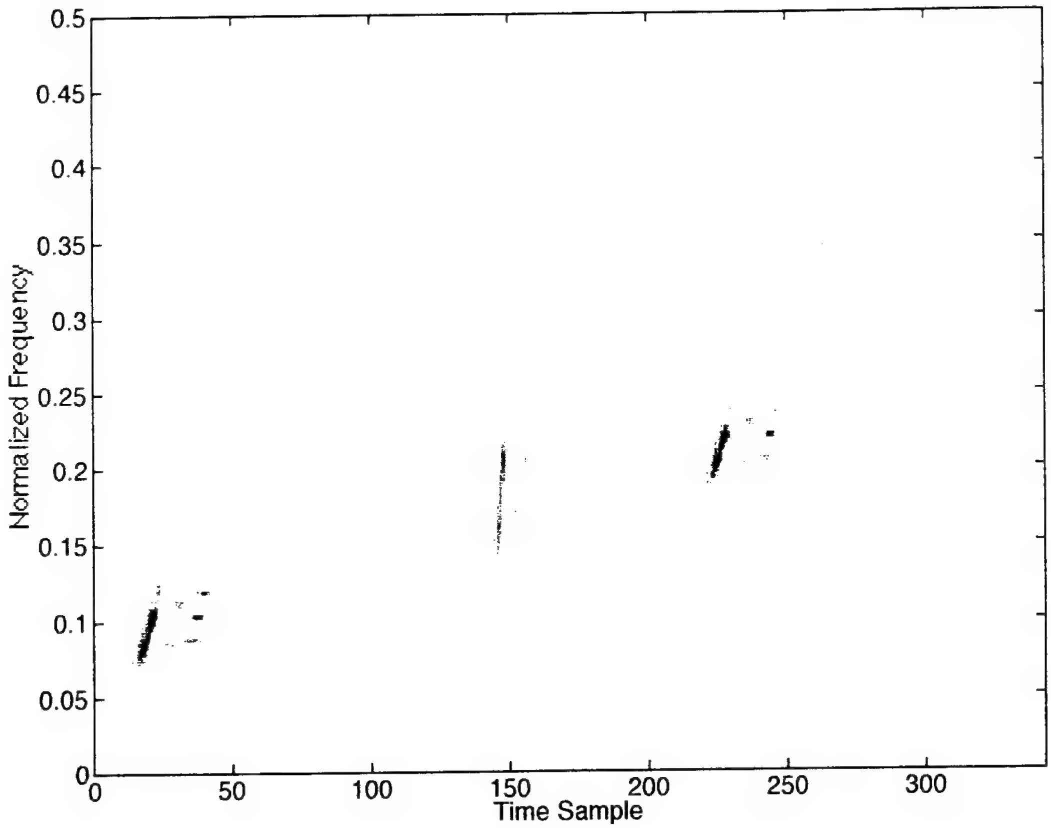


Figure 11: Three versions of a dolphin click: original, scaled and frequency shifted

impulsive and tonal components very well. It has scale invariance and reduced interference as well as many useful mathematical attributes.

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